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TRANSVERSE-MOMENTUM PARTON DENSITIES: GAUGE LINKS, DIVERGENCES AND SOFT FACTOR*

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We discuss the state-of-the-art of the theory of transverse-momentum dependent parton densities (TMD)s, paying special attention to their renormalization properties, the structure of the gauge links in the operator definition, and the role of the soft factor in the factorization formula within the TMD approach to the semi-inclusive processes. We argue that the use of the lightcone axial gauge offers certain advantages for a consistent definition of TMDs as compared to the off-the-light-cone gauges, or covariant gauges with off-the-lightcone gauge links.

Keywords: Parton distribution functions, Wilson lines, renormalization.

1. Introduction

The distribution functions of partons (in what follows we consider only quark distributions), depending on the longitudinal components x, as well as on the transverse components $\boldsymbol{k}_\perp$, of their momenta (hence TMD)s, accumulate useful information about the intrinsic motion of the hadron's constituents and enter as a nonperturbative input in the QCD approach

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to the semi-inclusive hadronic processes (see, e.g., Refs. [1–4]). The QCD factorization formula for a semi-inclusive structure function is expected to have the following symbolic form [5–7]

$$F(x_B, z_h, \boldsymbol{P}_{h\perp}, Q^2) = \sum_i \ e_i^2 \cdot H \otimes \mathcal{F}_D \otimes \mathcal{F}_F \otimes S \ , \tag{1}$$

where z_h and $\boldsymbol{P}_{h\perp}$ are the longitudinal and transverse fractions of the momentum of the produced hadron, respectively. This expression contains the hard (perturbatively calculable) part H, the (nonperturbative) distribution and fragmentation functions $\mathcal{F}_D$ and $\mathcal{F}_F$, and the soft part S. The latter is absent in the collinear (fully inclusive) picture, and will be discussed below. However, several problems arise in attempting to formulate the TMD approach in terms of the quantum field operators and their matrix elements: (i) Extra (rapidity) divergences appear already at the one-loop level, which invalidate the standard renormalization procedure [3,8–10]. (ii) A much more complicated (compared to the collinear case) structure of the gauge links leads to the non-universality of distribution or fragmentation functions (see, e.g., Refs. [11–13]). (iii) Several counter-examples have been given showing that the straightforward factorization formula (1) may fail, at least in some specific situations [14,15]. (iv) The role and explicit expression of the soft factor S can be different in different schemes. In what follows, we basically concentrate on the first and the last problem.

2. Divergences and renormalization properties of TMDs

The operator definition of the quark TMD (without the soft term) reads [5,6,9,10,16–21]

$$\begin{aligned}
&\tilde{\mathcal{F}}_{i/h}\left(x, \boldsymbol{k}_\perp\right) = \frac{1}{2} \int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{2\pi(2\pi)^2} \mathrm{e}^{-ik^+\xi^- + i\boldsymbol{k}_\perp \boldsymbol{\xi}_\perp} \\
&\times \Big\langle h|\bar{\psi}_i(\xi^-, \boldsymbol{\xi}_\perp)[\xi^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\xi}_\perp]^\dagger_{[n]}[\infty^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\infty}_\perp]^\dagger_{[\boldsymbol{l}]} \\
&\times \gamma^+[\infty^-, \boldsymbol{\infty}_\perp; \infty^-, \boldsymbol{0}_\perp]_{[\boldsymbol{l}]}[\infty^-, \boldsymbol{0}_\perp; 0^-, \boldsymbol{0}_\perp]_{[n]}\psi_i(0^-, \boldsymbol{0}_\perp)|h\Big\rangle \ .
\end{aligned} \tag{2}$$

This expression may be given a physical meaning, because it is formally gauge invariant. The gauge invariance is ensured by means of the path-ordered gauge links

$$\begin{aligned}
&[\infty^-, \boldsymbol{\xi}_\perp; \xi^-, \boldsymbol{\xi}_\perp]_{[n]} \equiv \mathcal{P} \exp\left[ig\int_0^\infty d\tau \ n^-_\mu \ A^\mu_a t^a(\xi + n^-\tau)\right] \ , \\
&[\infty^-, \boldsymbol{\infty}_\perp; \infty^-, \boldsymbol{\xi}_\perp]_{[\boldsymbol{l}]} \equiv \mathcal{P} \exp\left[ig\int_0^\infty d\tau \ \boldsymbol{l} \cdot \boldsymbol{A}_a t^a(\boldsymbol{\xi}_\perp + \boldsymbol{l}\tau)\right] \ ,
\end{aligned} \tag{3}$$

where we distinguish between longitudinal (lightlike, $n^2 = 0$) $[...]_{[n]}$ and transverse $[...]_{[\boldsymbol{l}]}$ links. (A generalized definition, which includes into the Wilson lines the spin-dependent Pauli term $F^{\mu\nu}[\gamma_\mu, \gamma_\nu]$, was recently worked out in Ref. [22]).

Of course, the above expressions have to be quantized, using, for instance, functional-derivative techniques. This means that the gluon potential in the gauge link has to be Wick contracted with corresponding terms in the interaction Lagrangian, accompanying the Heisenberg fermion (quark) field operators.

At the tree-level, the "distribution of a quark in a quark" (here we consider only ultraviolet (UV) and rapidity divergences, which are independent of the particular hadronic state) is normalized as

$$\tilde{\mathcal{F}}^{(0)}_{q/q}(x, \boldsymbol{k}_\perp) = \frac{1}{2} \int \frac{d\xi^- d^2\xi_\perp}{2\pi(2\pi)^2} \mathrm{e}^{-ik^+\xi^- + ik_\perp \cdot \xi_\perp}$$
$$\times \langle p | \bar{\psi}(\xi^-, \xi_\perp) \gamma^+ \psi(0^-, 0_\perp) | p \rangle = \delta(1-x)\delta^{(2)}(\boldsymbol{k}_\perp) \ , \tag{4}$$

and, formally, the integration over $\boldsymbol{k}_\perp$ yields the usual collinear (integrated) PDF

$$\int dk_\perp^{(2)} \tilde{\mathcal{F}}^{(0)}_{q/q}(x, \boldsymbol{k}_\perp) = F^{(0)}_{q/q}(x) = \delta(1-x) \ . \tag{5}$$

However, already in the calculation of the one-gluon contributions, one encounters—besides the normal UV divergences—certain pathological singularities. Namely, one has at the one-loop level the following singular terms:

(1) Standard UV poles $\sim \frac{1}{\varepsilon}$ in the dimensional regularization: they can be removed by the usual $R-$operation and are controlled by renormalization-group evolution equations (analoguous to the DGLAP equation in the integrated case).
(2) Pure *rapidity divergences*: they give rise to logarithmic and double-logarithmic terms of the form $\sim \ln \eta \ , \ \ln^2 \eta$. These terms, although they depend on the additional rapidity parameter η [1,5,6,8], do not affect the UV renormalization properties and can be safely resummed, e.g., by means of the Collins-Soper equation.
(3) Pathological *overlapping divergences*: they contain the UV and rapidity poles simultaneously $\sim \frac{1}{\varepsilon} \ \ln \eta$ and are considered to be *highly problematic*. The reason is that they prevent the removal of *all* UV-singularities by the standard $R-$procedure. Therefore, a special generalized renormalization procedure is needed in order to take care of those terms and enable the construction of well-defined renormalizable TMDs.

It is interesting to note that working in the lightcone gauge with the Mandelstam-Leibbrandt prescription [23,24], one doesn't get any overlapping divergences, at least in the leading loop order [25]. The renormalization properties of operators and matrix elements containing Wilson lines and loops with or without obstructions have been extensively studied in various situations—see, e.g., Refs. [26–30]. The specifics of the TMD consist in the fact that, though the fermion fields are separated by a spacelike distance, the gauge links lay on pure lightlike rays, or on the $2D-$transverse plane at lightcone infinity.

The analysis of the one-loop anomalous dimension of the TMD, given by Eq. (2), shows that the contribution of the overlapping singularity is nothing else, but the *cusp anomalous dimension* [29]. Therefore, in order to renormalize expression (2), one can apply, apart from the standard $R-$operation, an additional renormalization factor, which depends on the cusp angle and can be written as a vacuum matrix element of the Wilson lines evaluated along a special contour with an obstruction (cusp); viz.,

$$Z_\chi^{-1} = \left\langle 0 \left| \mathcal{P} \exp \left[ig \int_\chi d\zeta^\mu \ t^a A^a_\mu(\zeta) \right] \right| 0 \right\rangle \ . \tag{6}$$

The UV singularity of this factor cancels the cusp anomalous dimension from the overlapping divergence, thus rendering the re-defined TMD (2) renormalizable [9,10]. Therefore, the generalized renormalization procedure for the TMD can be formulated as

$$\tilde{\mathcal{F}}_{\rm ren}(x, \boldsymbol{k}_\perp, \chi, ...) = Z_{\rm R} \cdot Z_\chi \cdot \tilde{\mathcal{F}}(x, \boldsymbol{k}_\perp, \chi, ...) \ , \tag{7}$$

where $Z_{\rm R}$ is the usual renormalization constant, while Z_χ can be included in the definition of the TMD itself. In that case, it is treated as a "soft factor":

$$Z_\chi \equiv [\text{Soft Factor}] \ , \tag{8}$$

which is defined as

$$[\text{Soft Factor}] = \\ \langle 0 | \ \mathcal{P} {\rm e}^{ig \int_{\mathcal{C}_\chi} d\zeta^\mu \ t^a A^a_\mu(\zeta)} \cdot \mathcal{P}^{-1} {\rm e}^{-ig \int_{\mathcal{C}'\chi} d\zeta^\mu \ t^a A^a_\mu(\xi+\zeta)} | 0 \rangle \ , \tag{9}$$

where the contours $\mathcal{C}$ and $\mathcal{C}'$ are explicitly given in Ref. [10]. Therefore, the

generalized definition of the TMD reads [9,10,25]

$$\begin{aligned}
&\mathcal{F}_{i/h}\left(x, \boldsymbol{k}_\perp\right) = \frac{1}{2} \int \frac{d\xi^- d^2 \boldsymbol{\xi}_\perp}{2\pi(2\pi)^2} \mathrm{e}^{-ik^+\xi^- + i\boldsymbol{k}_\perp \cdot \boldsymbol{\xi}_\perp} \\
&\times \left\langle h \middle| \bar{\psi}_i(\xi^-, \boldsymbol{\xi}_\perp)[\xi^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\xi}_\perp]^\dagger_{[n]} [\infty^-, \boldsymbol{\xi}_\perp; \infty^-, \boldsymbol{\infty}_\perp]^\dagger_{[\boldsymbol{l}]} \right. \\
&\left. \times \gamma^+ [\infty^-, \boldsymbol{\infty}_\perp; \infty^-, \boldsymbol{0}_\perp]_{[\boldsymbol{l}]} [\infty^-, \boldsymbol{0}_\perp; 0^-, \boldsymbol{0}_\perp]_{[n]} \psi_i(0^-, \boldsymbol{0}_\perp) \middle| h \right\rangle \\
&\times \left[\text{Soft Factor}\right]. \qquad (10)
\end{aligned}$$

This function is free (at least, at the one-loop order) of pathological divergences and is a well-defined renormalizable quantity.

3. Factorization and role of the soft factor

The soft factor, introduced above, naturally enters in the factorization formula (1): [Soft Factor] $= S$. However, its interpretation is twofold.

On the one hand, it formally looks similar to the "intrinsic" Coulomb phase found by Jakob and Stefanis [31] in QED for Mandelstam charged fields involving a gauge contour which is a timelike straight line. The name "intrinsic" derives from the fact that this phase is different from zero even in the absence of external charge distributions. Its origin was ascribed in [31] to the long-range interaction of the charged particle with its oppositely charged counterpart that was removed "behind the moon" after their primordial separation. Note that the existence of a balancing charge "behind the moon" was postulated before by several authors—see [31] for related references—in an attempt to restore the Lorentz covariance of the charged sector of QED. This phase is acquired during the parallel transport of the charged field along a timelike straight line from infinity to the point of interaction with the photon field and is absent in the local approach, i.e., for local charged fields joined by a connector. It is different from zero only for a Mandelstam field with its own gauge contour attached to it and keeps track of its full history since its primordial creation. Keep in mind that the connector is introduced ad hoc in order to restore gauge invariance and is not part of the QCD Lagrangian. In contrast, when one associates a distinct contour with each quark field, one, actually, implies that these Mandelstam field variables should also enter the QCD Lagrangian (see [31] for more details). However, a consistent formulation of such a theory for QCD is still lacking and not without complications of its own.

The analogy to our case is the following. First, formally adopting a direct contour for the gauge-invariant formulation of the TMD in the light-cone gauge, the connector gauge link does not contribute any anomalous

dimension—except at the endpoints; this anomalous dimension being, however, irrelevant for the issue at stake. Hence, there is no intrinsic Coulomb phase in that case. Second, splitting the contour and associating each branch to a quark field, transforms it into a Mandelstam field and, as a result, adding together all gluon radiative corrections at the one-loop order, a η-dependent term survives that gives rise to an additional anomalous dimension. We have shown that this extra anomalous dimension can be viewed as originating from a contour with a discontinuity in the four-velocity $\dot{x}(\sigma)$ at light-cone infinity—a cusp obstruction.

Classically, it is irrelevant how the two distinct contours are joined, i.e., smoothly or by a sharp bend. But switching on gluon quantum corrections, the renormalization effect on the junction point reveals that the contours are not smoothly connected, but go instead through a cusp [10]. Here, we have a second analogy to the QED case discussed above. Similarly to the "particle behind the moon", this cusp-like junction point is "hidden" and manifests itself only through the path-dependent phase after renormalization.

In that case, the soft factor looks like an intrinsic property of the gauge-invariant operators containing the fermion fields and must be taken into account in order to construct consistently the gauge-invariant renormalizable two-particle matrix elements.

On the other hand, the soft factor appears as the result of the separation the "soft" contributions from the one-loop graphs [7]. In this situation it is needed in order to avoid the double-counting in the factorization formula (1). In general, these two soft factors might be different. In particular, it has been shown in Ref. [32] that the anomalous dimensions of the TMD within different subtraction schemes of the soft factors are different. The relationship between these frameworks will be studied elsewhere.

4. Conclusions

The above mentioned results have been obtained by using the lightcone axial gauge with a proper regularization of the gluon propagator [10,25]. The extra rapidity divergences can be treated by different methods, e.g., one may shift the gauge links off the lightcone, or use, alternatively, the off-the-lightcone axial gauge [5–7]. In these cases, the additional rapidity variable parameterizes the deviation of the gauge links from the light rays in terms of the axial-gauge fixing vector. An advantage of the "pure lightcone" frameworks is the more straightforward physical interpretation of the factorization and the role of the collinear Wilson lines in the definition of TMDs, as well as the direct relationship between the (unintegrated) TMDs

and the (integrated) collinear PDFs: one can get the collinear PDF, satisfying the DGLAP evolution equation, by simple $\boldsymbol{k}_\perp$-integration. In contrast, the "off-the-lightcone" frameworks don't allow us to perform such a procedure, so that more sophisticated methods must be invented. The complete proof of the QCD factorization, within the TMD approach (in particular within the "pure lightcone" scheme), as well as the clarification of the role played by the soft factors in different approaches, are still lacking.

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